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# A PHYSICALLY BASED CORRELATION FOR DROP SIZE IN ANNULAR FLOW

W. AMBROSINI,[1] P. ANDREUSSI[1] and B. J. AZZOPARDI[2]

[1]Department of Chemical Engineering, University of Pisa, Via Diotisalvi 2, 56100 Pisa, Italy

[2]Department of Chemical Engineering, University of Nottingham, Nottingham, England

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**Abstract**—The derivation of a correlation for drop size in annular flow based on a mechanistic model is presented. Optimum values of four constants were obtained by a fit to measured data over a wide range of gas and liquid flow rates, physical properties and pipe diameter.

*Key Words:* annular flow, drop size, film thickness, interfacial friction

## 1. INTRODUCTION

The configuration or flow pattern taken up by flowing gas–liquid mixtures depends on the flow rates of the two phases, the physical properties and the pipe geometry. Annular flow is one of the most important of these flow patterns; it is of particular interest as it occurs in a wide range of industrial equipment, such as nuclear reactor cores and steam generators, reboilers and condensers, in chemical plants and in oil and gas wells.

Annular flow is characterized by the presence of a liquid film flowing along the pipe wall, whilst a gas core containing dispersed droplets flows along the centre of the pipe. Knowledge of the fraction of liquid entrained in the gas stream is essential for the prediction of the main flow parameters. This fraction will be most correctly evaluated from models based on mechanisms by which droplets of different sizes are created at the gas–liquid interface and then deposit onto the liquid film flowing along the walls.

Some of the equations proposed for the prediction of drop diameter have been based on models of the main physical phenomena. Taylor (1940) studied the growth of a sinusoidal wave train on a stationary viscous medium of infinite extent and depth, proposing that the size of the entrained droplets should be proportional to the wavelength of the atomizing wave. He also suggested that the most frequently observed drop size would come from the fastest growing wave. These arguments led to the following relationship:

$$\frac{\rho_G U_G^2 d}{\sigma} = F\left(\frac{\rho_L}{\rho_G}\frac{\sigma^2}{\mu_L^2 U_G^2}\right), \qquad [1]$$

in which $\rho_G$, $U_G$, $d$ and $\sigma$ are, respectively, the gas density, gas velocity, droplet diameter and liquid surface tension, and $\rho_L$ and $\mu_L$ are the liquid density and dynamic viscosity, respectively. Tatterson (1975) and Tatterson *et al.* (1977) modified Taylor's theory in order to account for the finite depth of the liquid layer. These authors assumed that the wavenumber giving the greatest contribution to the rate of atomization varies inversely with the height of the liquid film. Then, on the basis of a force balance on a developing ligament at the gas–liquid interface, they proposed the following expression for droplet diameter:

$$\frac{d}{m} = \mathrm{A}\mathrm{We}_m^{-0.5} = \mathrm{A}\left(\frac{\rho_G U_G^{*2} m}{\sigma}\right)^{-0.5}, \qquad [2]$$

where A is a real constant, $\mathrm{We}_m$ is the Weber number based on the film thickness, $m$, and the gas friction velocity, $U_G^*$, is defined as

$$U_G^* = \sqrt{\frac{\tau_i}{\rho_G}}, \qquad [3]$$

where $\tau_i$ is the interfacial shear stress.

The gas friction velocity was used in the Weber number because Tatterson *et al.* (1977) argued that it best represented the effect of the gas flow close to the surface of the atomizing waves.

Correlations for drop size have also been proposed by other authors. Andreussi & Zanelli (1979), correlating their own data, showed that droplet diameters can be represented by an equation similar to the one proposed by Tatterson (1975) [2], but using $-1/3$ exponent for the Weber number. An alternative approach was developed by Azzopardi *et al.* (1980) and, more recently, by Lopes & Dukler (1985), who assumed that the drop size is governed by resonance phenomena related to gas-phase turbulence. From this model, the droplet diameter is found to be proportional to a Weber number based on the pipe diameter, $\mathrm{We}_D$, raised to the power of $-0.6$. This Weber number is defined as

$$\mathrm{We}_D = \frac{\rho_G U_G^2 D}{\sigma}, \qquad [4]$$

where $D$ is the pipe diameter.

However, as break-up occurs soon after removal from the wall film, but before the liquid has accelerated to the gas velocity where turbulence can take effect, this type of resonance is not expected to be important.

In the correlations of Azzopardi *et al.* (1980) and of Azzopardi (1985), the effect of droplet coalescence in the gas core due to collisions is taken into account through a term proportional to the concentration of entrained liquid.

Recent experimental investigations of the effect of fluid properties on drop size and other flow parameters (Jepson *et al.* 1989, 1990; Willetts 1987) have provided a new basis for assessing earlier correlations. With regard to droplet sizes, it has been noted (Jepson *et al.* 1989) that the new information does not agree with the prediction of the equations of both Azzopardi *et al.* (1980) and Azzopardi (1985).

The trends highlighted by the new data have been utilized in the present work in order to develop a correlation for the Sauter mean diameter of the droplets. The approach proposed by Tatterson (1975) is followed and, therefore, [2] is taken as the starting point of the correlation, since it represents with acceptable accuracy data from the highest gas velocity. Multiplying factors have then been added to account for phenomena not considered in the original development. Equation [2] requires values of the film thickness and the interfacial friction factor (the latter is used in the definition of the friction velocity). Published correlations for these two parameters have been reviewed critically and, where appropriate, modified to provide an acceptable fit of the new data as well as earlier data.

## 2. EXPERIMENTAL DATA CONSIDERED

In the present work, extensive use has been made of a data bank on annular flow created at Harwell Laboratory. This bank contains data on entrained fraction, pressure gradient and film thickness for vertical upflow in round tubes. It is convenient to group the data as old and new data. The former consists of the data used by Whalley & Hewitt (1978) in their work on entrainment correlations, the latter includes more recent data by Willetts (1978), Asali (1984), Leman (1985) and Owen (1986).

Only the newer data by Willetts, Asali and Leman and older data by Gill *et al.* (1964, 1969) and Whalley *et al.* (1973) (Gill & Whalley in the following) include measured film thicknesses.

The data by Willetts (1987) and Asali (1984) are from different fluid systems (air–water, helium–water, air–sulpholane, air–1,1,1-trichloroethane and air–fluoroheptane were used by Willetts; air–water and air–glycerine were used by Asali) and different diameters (10.26 mm in the data by Willetts; 42 and 23 mm in the data by Asali). From this, the effects of gas density, surface tension, liquid viscosity and density and pipe diameter on the measured values can be evaluated. The data by Leman were from an air–glycerine system with pipe i.d. = 42 mm, whilst that of Gill & Whalley were from air–water flowing in an i.d. = 31.8 mm pipe.

The same data were used to develop the correlation for the interfacial friction factor, since in this case, measured values of film thickness were required as well as pressure gradients.

Table 1. Range of flow parameters and fluid properties in the experimental data considered in setting up the proposed correlations

| Flow parameter or fluid property | Correlation for $m$ | Correlation for $f_i$ | Correlation for $d_{32}$ |
|---|---|---|---|
| Pipe diameter [mm] | 10.26–42.20 | 10.26–42.20 | 10.26–32.00 |
| Liquid mass flux [kg/m$^2$ s] | 1.2–710.1 | 1.5–182.4 | 15.0–300.0 |
| Gas mass flux [kg/m$^2$ s] | 9.8–235.5 | 9.8–129.5 | 12.0–140.0 |
| Entrance mass flux [kg/m$^2$ s] | 0.0–572.4 | 0.0–147.6 | 0.0–178.4 |
| Liquid density [kg/m$^3$] | 1000.0–1745.0 | 1000.0–1745.0 | 998.3–1307.0 |
| Gas density [kg/m$^3$] | 0.27–4.20 | 0.27–2.99 | 0.27–2.41 |
| Liquid viscosity [kg/ms] | $0.9 \times 10^{-3}$–$5.9 \times 10^{-3}$ | $0.9 \times 10^{-3}$–$5.9 \times 10^{-3}$ | $1.0 \times 10^{-3}$–$1.3 \times 10^{-3}$ |
| Gas viscosity [kg/ms] | $1.8 \times 10^{-5}$–$1.9 \times 10^{-5}$ | $1.8 \times 10^{-5}$–$1.9 \times 10^{-5}$ | $1.8 \times 10^{-5}$–$1.9 \times 10^{-5}$ |
| Surface tension [N/m] | $1.3 \times 10^{-2}$–$7.3 \times 10^{-2}$ | $1.3 \times 10^{-2} - 7.3 \times 10^{-2}$ | $2.5 \times 10^{-2}$–$7.3 \times 10^{-2}$ |
| Film thickness [mm] | 0.043–0.737 | 0.028–0.367 | — |
| Drop Sauter diameter [mm] | — | — | 0.010–0.180 |

A separate bank was created for droplet size information. This included data from Jepson *et al.* (1989, 1990), Teixeira (1988), Andreussi *et al.* (1978) and Azzopardi *et al.* (1980). The data by Jepson *et al.* were taken on a small diameter tube (10.26 mm) and with air–water, helium–water and air–1,1,1-trichloroethane. The effect of gas density on the measured Sauter mean diameter, $d_{32}$, was examined by using air and helium as the gas phase; 1,1,1-trichloroethane has been used to test the effect of surface tension, its surface tension is about one-third that of water. Tubes with larger diameters were used in the tests by Teixeira (32 mm), Azzopardi *et al.* (32 mm) and Andreussi *et al.* (24 mm); in these tests only air–water was considered. Jepson *et al.*, Teixeira and Azzopardi *et al.* made use of the laser diffraction technique developed by Swithenbank *et al.* (1976) for droplet size measurement; Andreussi *et al.* employed a microphotographic technique.

Table 1 summarizes the main variable ranges involved in the data considered in setting up or validating each of the proposed correlations for $m$, $f_i$ and $d_{32}$.

## 3. PREDICTION OF THE MAIN FLOW PARAMETERS

*Film thickness*

Most of the available correlations for film thickness take the form

$$m_L^+ = \mathrm{X}\mathrm{Re}_{\mathrm{LF}}^{\mathrm{Y}}, \qquad [5]$$

where X and Y are constants. The dimensionless thickness, $m_L^+$, and the liquid film Reynolds number, $\mathrm{Re}_{\mathrm{LF}}$, are defined as

$$m_L^+ = \frac{mU_L^*}{\nu_L} \qquad [6]$$

and

$$\mathrm{Re}_{\mathrm{LF}} = \frac{G_{\mathrm{LF}}D}{\mu_L}, \qquad [7]$$

where $\nu_L$ and $G_{\mathrm{LF}}$ are the liquid kinematic viscosity and the liquid film mass flux, respectively. The liquid friction velocity, $U_L^*$, required for the calculation of the dimensionless film thickness is given by

$$U_L^* = \sqrt{\frac{\tau_c}{\rho_L}}, \qquad [8]$$

where $\tau_c$ is a characteristic shear stress which, at high gas velocities, may be identified with both the wall or the interfacial shear stress. Henstock & Hanratty (1976) suggested a definition of $\tau_c$ for thin films to be

$$\tau_c = \tfrac{1}{3}\tau_i + \tfrac{2}{3}\tau_w, \qquad [9]$$

where $\tau_i$ and $\tau_w$ are the interfacial and wall shear stresses, respectively. This description, which requires the separate evaluation of the interfacial and wall shear stresses from measured pressure drop and film thickness, has been adopted in this work for the calculation of the dimensionless film thickness. However, it was verified that the difference between $\tau_c$ and $\tau_i$ and $\tau_w$ is negligible for most of the data points considered. The shear stresses can be calculated using the following relationships derived from momentum balances relative to vertical upward flow:

$$\tau_w = \frac{D}{4}\left[\left|\frac{dp}{dz}\right| - (\alpha_{LF}\rho_L + \alpha_{GC}\rho_{GC})g - \alpha_{GC}\rho_{GC}U_{GC}\frac{dU_{GC}}{dz}\right] \tag{10}$$

and

$$\tau_i = \frac{D_i}{4}\left[\left|\frac{dp}{dz}\right| - \rho_{GC}g - \rho_{GC}U_{GC}\frac{dU_{GC}}{dz}\right], \tag{11}$$

where $dp/dz$ is the pressure gradient, $D_i = D - 2m$ is the film internal diameter, $\alpha_{LF}$ and $\alpha_{GC}$ are the volume fractions occupied by the liquid film and by the gas and the entrained droplets, respectively, $g$ is gravitational accelaration and the gas core density and velocity are defined as

$$\rho_{GC} = \frac{G_G + G_{LE}}{\dfrac{G_G}{\rho_G} + \dfrac{G_{LE}}{\rho_L}} \tag{12}$$

and

$$U_{GC} = \frac{1}{\alpha_{GC}}\left(\frac{G_G}{\rho_G} + \frac{G_{LE}}{\rho_L}\right), \tag{13}$$

$G_C$ and $G_{LE}$ being the mass fluxes of the gas and of the entrained liquid, respectively.

A number of values have been published for the constants X and Y in [5] (Henstock & Hanratty 1976; Asali *et al.* 1985; Kosky 1971). In the present work, the available correlations have been reviewed in order to select the most appropriate correlation to be used for the evaluation of film thickness in the equation for droplet size prediction.

Figure 1 shows the results obtained for the data of Willetts (all fluids), Asali (air–water and air–glycerine with $D = 42$ mm) and Leman (air–glycerine), together with the higher Reynolds number data by Whalley & Gill referred to in section 2. It can be seen that the "lower Reynolds number" data agree closely with the law proposed by Asali *et al.* (1985):

$$m_L^+ = 0.34\,\mathrm{Re}_{LF}^{0.6}. \tag{14}$$

Although the data of Whalley give a large scatter at intermediate Reynolds numbers, a change in the slope is apparent for $\mathrm{Re}_{LF} \gtrsim 1000$.

It was found that data above $\mathrm{Re}_{LF} > 1000$ are well-predicted by an equation based on the "one-seventh power law", proposed by Kosky (1971):

$$m_L^+ = 0.0512\,\mathrm{Re}_{LF}^{0.875}. \tag{15}$$

Willets (1987) reports that this law gives also reasonable predictions of the high-pressure steam–water data taken by Wurtz (1978).

*Interfacial friction factor*

The evaluation of gas friction velocity, $U_G^*$, appearing in $\mathrm{We}_m$ involves the calculation of the interfacial shear stress (see [3]). Most authors define $\tau_i$ in terms of an interfacial friction factor, $f_i$, representing the roughness of the gas–liquid interface:

$$\tau_i = \tfrac{1}{2}f_i\rho_G U_G^2. \tag{16}$$

The interfacial shear stress is often calculated from the measured pressure drops, without taking explicitly into account momentum transfer due to the interchange of droplets between the liquid film and gas core.

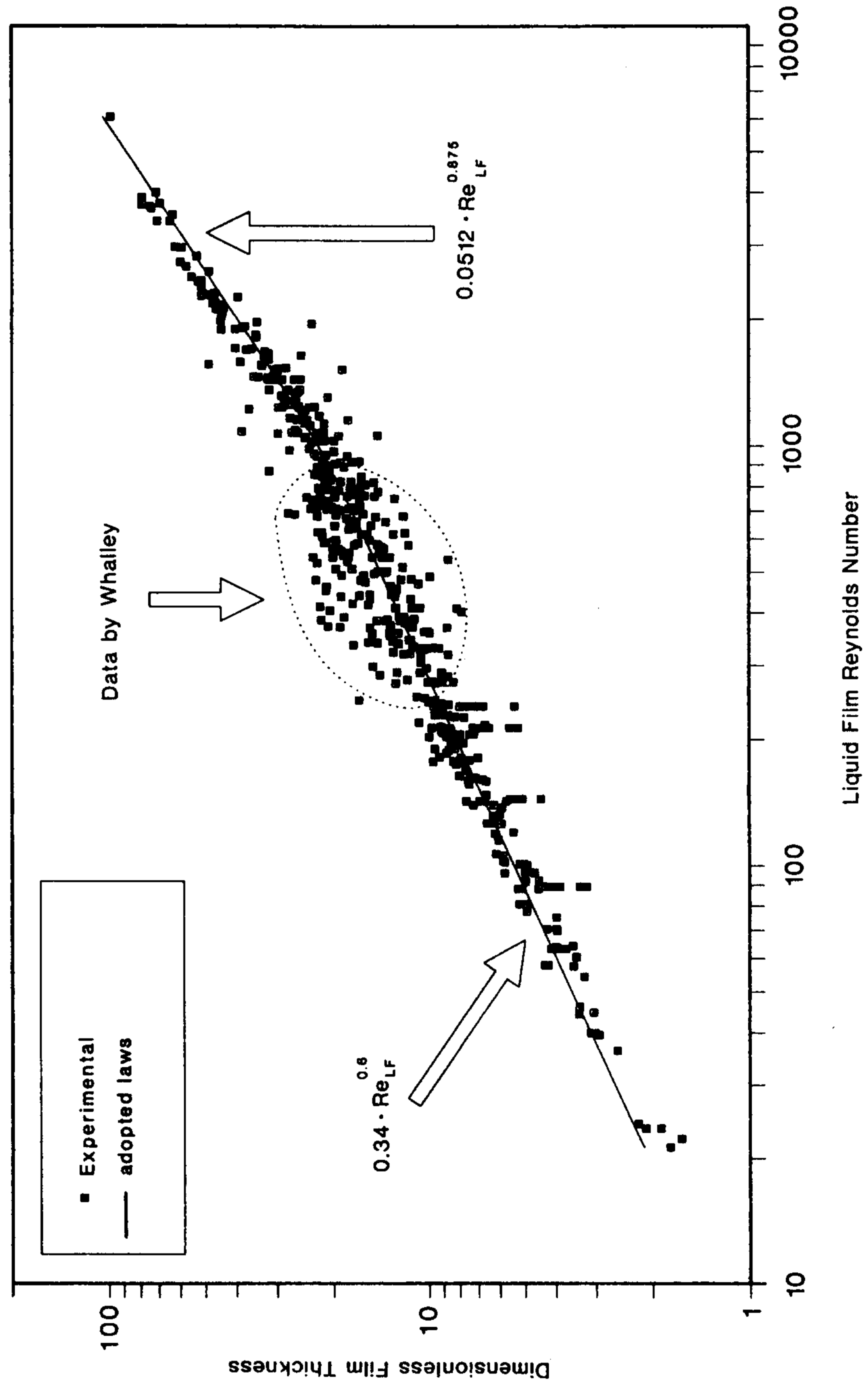


Figure 1. Film thickness prediction: all considered data sources.

Experimental observations show that:

- Even without entrainment, the measured pressure drop is greater than it would be if the gas core was flowing in a smooth pipe; this leads to the conclusion that waves and ripples formed at the gas–liquid interface can be considered as an increased roughness with respect to the smooth pipe condition.
- The interfacial friction factor decreases with decreasing film thickness; below a lower limit for $m$, $f_i$ is equal to the smooth pipe friction factor, $f_s$.
- A linear dependence of $f_i$ on the film thickness is very often observed.

A number of relationships which take into account the experimental observation mentioned above have been proposed for $f_i$ (Wallis 1969; Asali *et al.* 1985; Willetts 1987).

In the present work an attempt has been made to characterize $f_i$ on the basis of the formulation proposed by Asali (1985) and tested also by Willetts (1987), rephrased in the form

$$\frac{f_i}{f_s} = 1 + \mathrm{K}\,\mathrm{We}_D^p\,\mathrm{Re}_G^q(m_G^+ - m_{GO}^+), \qquad [17]$$

where K, p and q are real constants, $m_{GO}^+$ is a function of fluid properties and $\mathrm{Re}_G$ and $m_G^+$ are the gas and the roughness Reynolds numbers given by the relationships

$$\mathrm{Re}_G = \frac{U_G D}{\nu_G} \qquad [18]$$

and

$$m_G^+ = \frac{m U_G^*}{\nu_G}, \qquad [19]$$

in which $\nu_G$ is the gas kinematic viscosity. The smooth pipe friction factor, $f_s$, is finally evaluated by the correlation

$$f_s = 0.046\,\mathrm{Re}_G^{-0.2}. \qquad [20]$$

In [17], $f_i$ is assumed to be a function of both $\mathrm{Re}_G$ and $m_G^+$. This is equivalent to assuming that $f_i$ is a function of $m_G^+$ and $m/D$. It follows that [17] should be a more general correlation than the equations proposed by Wallis (1969) and by Asali *et al.* (1985). The dependence on $\mathrm{We}_D$ can be related to the effect of surface tension on the size of interfacial roughness (e.g. Willetts 1987).

Optimum values of the coefficients in [17] have been determined by a least-square fit of the collected data for which $U_G > 25$ m/s. At lower gas velocities, the effect of gravity on the interfacial structure becomes appreciable. An additional dimensional group involving gravity is then required. For $m_{GO}^+$ a function of fluid properties only was sought.

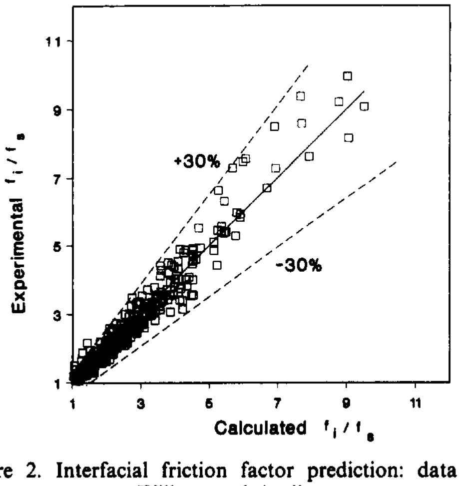


Figure 2. Interfacial friction factor prediction: data by Willetts and Asali.

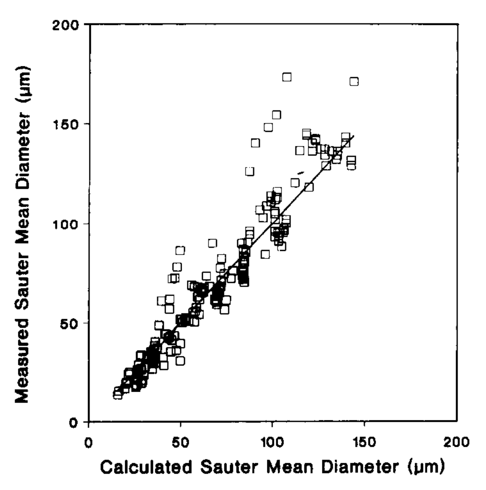


Figure 3. Drop size predictions by [26]: all considered data sources.

The result of the above is

$$\frac{f_i}{f_s} = 1 + 13.8\,\mathrm{We}_D^{0.2}\,\mathrm{Re}_G^{-0.6}\left(m_G^+ - 200\sqrt{\frac{\rho_G}{\rho_L}}\right). \qquad [21]$$

Figure 2 shows the plot of predicted vs measured values of $f_i$. It can be seen that the proposed equation gives predictions within $\pm 30\%$ of the experimental data.

## 4. PROPOSED DROP SIZE CORRELATION

The form selected for the equation for drop size is that shown in [2], due to Tatterson, in which $m$ and $f_i$ are calculated by the correlations developed in the last section ([14], [15] and [21]). However, this equation is inadequate as it stands, as experimental data showed some further effects that need to be taken into account.

The scatter in prediction can be reduced considerably by including the density ratio $\rho_G/\rho_L$ raised to a power of 0.5–0.7 in [2]. This factor was also used in the correlations of, for example, Azzopardi *et al.* (1980) and Nigmatulin *et al.* (1986). It is also visible in the data trends.

Tatterson's equation is based on the phenomena of ligament formation and detachment at the gas–liquid interface. Coalescence of droplets in the gas core is not accounted for. This latter phenomenon, the result of collisions between droplets, leads to an increase in the Sauter mean diameter which is proportional to the droplet concentration. In this work, this effect has been modelled by considering the possibility of droplet collision.

For the sake of simplicity, it is assumed that all the droplets have the same diameter and that the time-averaged axial velocity is the same as that of the gas. Their random motion can then be treated in a manner similar to gas kinetic theory by defining a mean-free-path between collisions:

$$\lambda_c = \frac{C}{d^2 N}, \qquad [22]$$

where $N$ is the number of droplets per unit volume, given by

$$N = \frac{G_{LE}}{\rho_L U_G}\frac{6}{\pi d^3}, \qquad [23]$$

and the real constant C depends on the assumptions made in calculating the droplet relative velocity. The probability that a single droplet has not yet experienced any collisions after having travelled an overall distance $L$ in the gas core can be expressed as

$$P_{nc} = \exp\left(-\frac{L}{\lambda_c}\right). \qquad [24]$$

Then, it is assumed that the length $L$ scales as the pipe diameter and the increase in droplet diameter due to coalescence is inversely proportional to $P_{nc}$ to some power. This assumption can be explained using the simplified argument that at each collision the droplet increases its volume of a quantity equal to its initial volume. Therefore, after having travelled a distance $L$ the average diameter of a population of equal droplets is increased by a factor

$$\sqrt[3]{1 + \frac{L}{\lambda_c}}. \qquad [25]$$

By approximating $1 + L/\lambda_c$ with an exponential, the previously mentioned result is obtained. The use of an exponential as correction factor facilitates the optimization of free parameters by a linear least-square method.

Introducing the correction factors described above leads to a new version of [2] expressed in terms of the Sauter mean diameter, $d_{32}$:

$$\frac{d_{32}}{m} = x_1\left(\frac{\sigma}{\rho_G f_i U_G^2 m}\right)^{0.5}\left(\frac{\rho_G}{\rho_L}\right)^{x_2}\exp\left(x_3\frac{G_{LE}}{\rho_L U_G}\frac{D}{d_{32}}\right), \qquad [26]$$

in which $x_j (j = 1, 2, 3)$ are constants and $U_G$ is the actual gas velocity based on the cross-section of the gas core.

Fitting [26] to the experimental data yields values of 5.0, 0.56 and 0.36, respectively, for the three constants. The results obtained are plotted in figure 3. They show that predicted and measured drop sizes agree, although there is a consistent underprediction of some data. A closer examination of these discrepancies shows that the underprediction occurred at low gas velocities in the air–water and helium–water data of Jepson *et al.* (1989) taken from a small diameter pipe. This might be related to the different mechanisms of atomization reported by Azzopardi (1983).

In order to obtain a more widely based equation for drop sizes, the following arguments have been used. It appears to be reasonable to assume that at low $U_G$ large lumps of liquid are entrained by the gas. The final size of entrained droplets is then determined by the interaction between these lumps and the gas stream. This phenomenon has been analysed by Hinze (1949), who proposed a simple equation for drop size:

$$\mathrm{We}_d = \frac{\rho_G U_G^2 d}{\sigma} = \text{const.} \qquad [27]$$

It is assumed that [27] controls the drop size at low $U_G$. This equation can be combined with the correlation developed by Tatterson *et al.* (1977):

$$\begin{aligned}\frac{d_{32}}{m} &= \sqrt{[A_1(\mathrm{We}_m)^{-0.5}]^2 + \left(A_2 \frac{\sigma}{\rho_G U_G^2 m}\right)^2} \\ &= A_1(\mathrm{We}_m)^{-0.5}\sqrt{1 + A_3\left(\frac{\sigma f_i}{\rho_G U_G^2 m}\right)},\end{aligned} \qquad [28]$$

where $A_1$, $A_2$ and $A_3$ are constants. For simplicity, assuming as in a classical formulation proposed by Wallis (1969), which holds for thick films,

$$f_i \approx \frac{m}{D}, \qquad [29]$$

and using the approximation

$$\sqrt{(1+\xi)} \approx \sqrt{e^{\xi}} = e^{0.5\xi}, \qquad [30]$$

which allows us to deal with a correction factor in the form of an exponential, the final correlation can be formulated as

$$\frac{d_{32}}{m} = x_1\left(\frac{\sigma}{\rho_G f_i U_G^2 m}\right)^{0.5}\left(\frac{\rho_G}{\rho_L}\right)^{x_2}\exp\left(x_3 \frac{G_{LE}}{\rho_L U_G}\frac{D}{d_{32}} + \frac{x_4}{\mathrm{We}_D}\right). \qquad [31]$$

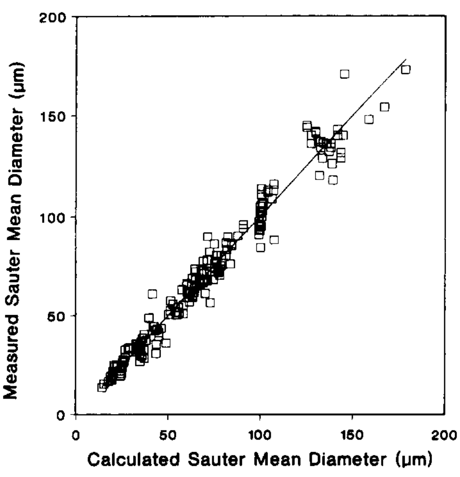


Figure 4. Drop size prediction by [31]: all considered data sources.

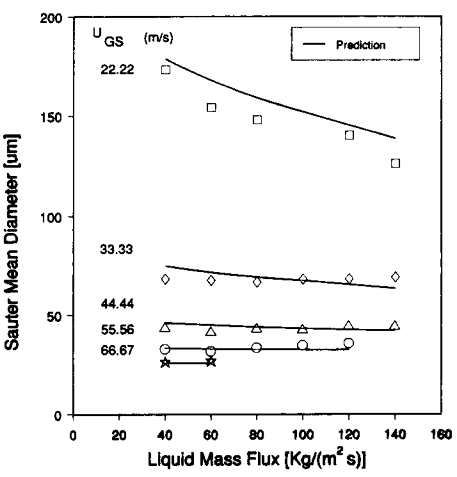


Figure 5. Drop size prediction by [31]: air-water data by Jepson *et al.* (1989).

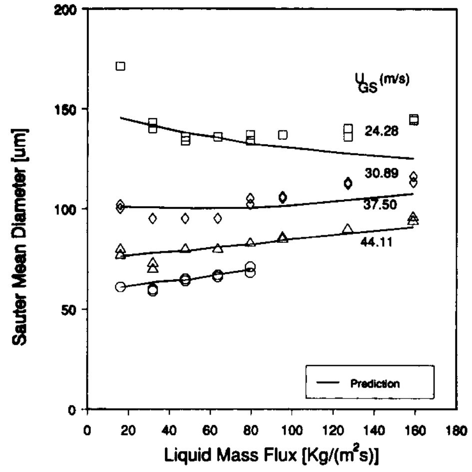


Figure 6. Drop size prediction by [31]: air–water by Azzopardi *et al.* (1980).

The optimized values of the four constants were found to be: $x_1 = 22.0$, $x_2 = 0.83$, $x_3 = 0.60$ and $x_4 = 99.0$.

Figure 4 shows the predictions for all the data considered. Comparison with figure 3 illustrates the efficacy of the last correction factor. Figures 5 and 6 are plots of the Sauter mean diameter against liquid flow rate, showing that [3] correctly predicts the effect of this parameter. In particular, the decrease in drop size with increasing $G_L$ at low gas and liquid flow rates is correctly predicted. This is probably due to the increase in the interfacial factor with film thickness.

## 5. SUMMARY AND CONCLUSIONS

In the present work a drop size correlation based on the model proposed by Tatterson *et al.* (1977) has been developed considering recently obtained experimental information on the effect of fluid properties.

The form of the correlation adopted requires appropriate relationships for film thickness and interfacial friction factor. Therefore, available correlations for these parameters have been considered and where appropriate, improved and tested against a data bank collected at Harwell Laboratory.

For film thickness, low liquid film Reynolds number data were found to be very well predicted by the correlation of Asali *et al.* (1985): $m_L^+ = 0.34\ \mathrm{Re}_{LF}^{0.6}$. Higher Reynolds number data ($\mathrm{Re}_{LF} > 1000$) were better represented by the one-seventh power law suggested by Kosky (1971).

Interfacial friction factor data were correlated using an extension of the approach proposed by Asali *et al.* (1985). The resulting equation is a fit to the data of Willetts (1987) and Asali (1984) who used different fluid properties and pipe diameters.

These correlations for film thickness and interfacial friction factor have been used in the drop size correlation developed by Tatterson (1975). Comparison of the prediction of this equation against the data of Jepson *et al.* (1989, 1990), Teixeira (1988), Andreussi *et al.* (1978) and Azzopardi *et al.* (1980) indicates that the equation requires a number of correction factors to allow for hitherto unaccounted for phenomena. These correlations refer to:

- A density ratio effect observed in the experimental data.
- Drop coalescence in the gas core due to collisions: the calculated Sauter mean diameter was divided by a term proportional to the probability of non-collision of the drop.
- An effect consisting of a considerable increase in drop size at low gas velocity. This effect has been accounted for by a Weber number based on the pipe diameter.

The correlation developed gives a good description of the observed experimental trends. In particular, the dependence on the liquid flow rate of the Sauter mean diameter at low liquid flow rate was well-predicted.

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